

Problem 3

Use the preliminary test to decide whether the following series are divergent or require further testing. *Careful:* Do *not* say that a series is convergent; the preliminary test cannot decide this.

$$\sum_{n=1}^{\infty} \frac{n+3}{n^2+10n}$$

Solution

Take the limit of the summand as $n \rightarrow \infty$.

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{n+3}{n^2+10n} &= \lim_{n \rightarrow \infty} \frac{\frac{1}{n} + \frac{3}{n^2}}{1 + \frac{10}{n}} \\ &= \frac{0+0}{1+0} \\ &= 0 \end{aligned}$$

Since it's zero, no conclusion can be drawn. Further testing is needed.